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VACUUM TUBES

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TO MY COLLEAGUES AND STUDENTS
PREFACE

This book is the outgrowth of a course in vacuum-tube design given for many years at Stanford University to senior and graduate students in electrical engineering and physics. It is concerned with the determination of vacuum-tube characteristics in terms of the electron action within the tube. The book attempts to bridge the gap between the physical laws that lie behind the electron behavior and the external characteristics of the tubes themselves.

It is hoped that the point of view taken will be acceptable to both physicists and engineers. The development of the physical laws involved is indicated, after which emphasis is placed upon their description and utilization. Although this book cannot pretend to give much design information, the attempt has been to include enough of the basic relations, physical data, and significant references to make it a useful reference source to vacuum experimenters and tube designers.

Vacuum tubes may seem a rather special subject to which to restrict the material in a book. Actually this is not so. In preparing the book so much material was collected that the contents had to be restricted to first-order effects. It is felt that although engineers and physicists working with vacuum tubes are primarily concerned with the utilization of already developed tubes, the successful application of these tubes is greatly enhanced by a knowledge of their limitations and an understanding of the origin of their characteristics. This is particularly true since there are many occasions when it is desired to use tubes under conditions different from those specified by the manufacturer. Under these conditions it is imperative to know how far one may depart from recommended operating conditions without exceeding some design limitation of the tube. This, in turn, requires a knowledge of how the tube operates.

Circuits and tube applications are so completely covered in the textbook and periodical literature that no effort has been made to include information on these subjects. Only in the case of ultra-high-frequency tubes where the tube cannot be completely separated from the circuit have circuit considerations been included.

The author is indebted to many people for assistance rendered in the preparation of this book. He is particularly indebted to Dr. F. E. Ter-
man, dean of the Stanford School of Engineering, who was a constant source of inspiration and encouragement, and who made many valuable suggestions and gave much direct assistance in checking the work. The author is also indebted to Prof. Paul Kirkpatrick, head of the Physics Department at Stanford, for suggestions on the material of Chaps. 3 to 6 and 9; to Prof. L. Marton for suggestions on the material of Chaps. 13 to 15 and 20; and to C. V. Litton for much information and suggestions relative to Chap. 21. He is indebted to Evelyn G. Sarson, who typed a large part of the manuscript in its final form. O. O. Pardee and Will Harman assisted in the correction of the entire work. Lastly, the author is more than a little indebted to his wife, who personally typed much of the manuscript and was a source of constant assistance.

KARL R. SPANGENBERG

Palo Alto, Calif.
January, 1948
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